Rough approximation of a preference relation by multi-decision dominance for a multi-agent conflict analysis problem

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\textbf{A B S T R A C T}

Multi-attribute group decision-making (MAGDM) has evoked increasing attention in recent years. Meanwhile, many valuable approaches have been developed to solve various MAGDM problems. In this paper, we consider a MAGDM problem in the presence of multi-attribute and multi-decision decision making with preference, namely the MA&MD decision problem. It involves the assignment of objects (actions), evaluated based on a set of conditional attributes, to pre-defined and preference-ordered multi-decision making. The actions are described by a finite set of conditional attributes and decision attributes. Both types of attribute take the values from their domain with preference order. In order to construct a comprehensive preference evaluation model that could be used to support the optimal choice task, we define two dominance relations, one on the condition attribute set and the other on the decision attribute set. We then present the lower and upper approximations of a preference relation defined by the decision attribute set based on a multi-decision preference dominance relation. Meanwhile, we propose an approach to decision making based on the rough set model established in this paper. The approach to decision making is derived from the lower approximation of decision classes with a preference dominance relation. The idea and decision rule are applied to solving a multi-agent conflict analysis decision problem. This method addresses limitations of the Pawlak conflict analysis model and thus improves on that model. Furthermore, to give practical significance to this management decision making approach, we present two extended models of the multi-decision preference dominance-based rough set as well as the corresponding decision making method. Moreover, we compare the proposed approach to previous studies of dominance-based rough set approaches to multiple attribute (criteria) decision making. The main contribution of this paper is two-fold. One is to establish a generalization of the classical dominance-based rough set approach, i.e., the model of multi-decision preference dominance-based rough set. Another is to present a new approach to deal with the multi-agent conflict analysis decision making problem based on the proposed multi-decision rough set approach.

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\section{1. Introduction}

Decision making is a procedure to find the best alternative among a set of feasible alternatives. The solution can be complex or simple. Because all decision-making problems have multiple alternatives and criteria, an increase in the number of
et al. [25] applied the dominance-based rough set approach to group decision making theory. They consider the decision approximations of partial preorders. Recently, Chakhar and Saad [9] study multicriteria classification problems in group decision evaluations given by multiple decision makers. Their extension also provides a general methodology for rough approximations of multiple decision makers by introducing specific concepts related to dominance with respect to minimal profiles of objects (or actions, schemas) evaluated based on a set of criteria (i.e., conditional attributes with a preference-ordered values set) to pre-defined and preference-ordered multi-decision making (i.e., multi-decision attributes with preference-ordered values sets, or multi-decision makers with preference-ordered values sets). It is assumed that there generally is a correlation between evaluation on conditional attributes and assignment to decision attributes with multi-decision makers, i.e., the better the evaluation of an action (object) on the conditional attributes, the better the assignment of an action (object) to the decision attributes (multi-decision makers) and vice versa.

The multiple decision with preference problem may arise in many real-world situations [21]. Let us consider, for example, the problem of the conflict analysis decision problem with multi-agents, in which objects (the favorable solution for a conflict situation) may be evaluated with regard to the disputes in a conflict situation (the conditional attribute) with “small,” “medium,” and “high” levels. Meanwhile, the favorable solution for a conflict situation may be evaluated for agents (the decision attribute) with preference values of “bad,” “average,” “good,” and “excellent.” That is, all agents related to a conflict situation are evaluated with regard to their individual preferences for every solution to the conflict situation. We consider the example of the Middle East conflict given in [14]. The following five issues, with slight modifications from [14], are the criteria (conditional attributes): autonomous Palestinian state in the West Bank and Gaza, Israeli military outpost along the Jordan River, Israeli retention of East Jerusalem, the Israeli military outposts on the Golan Heights, and Arab countries’ granting of citizenship to Palestinians who choose to remain within their borders. The six agents, Israel, Egypt, Palestine, Jordan, Syria, and Saudi Arabia, are the decision attributes. Obviously, the Middle East conflict is a multiple attribute and multiple decision problem. Similarly, the conflict problem of labor-management negotiations also entails a multiple attribute and multiple decision problem. Many mathematical models [52] have been proposed to investigate these multi-attribute and multi-decision problems from the viewpoint of conflict. The existing approaches to these problems are the conflict analysis methods based on the classical Pawlak rough set [43,44,53]. Though the existing approaches present expected solutions for these decision making problems, there also exist some limitations, as pointed out by Deja [14].

Conflict is, no doubt, one of the most characteristic attributes of human nature, and therefore the study of conflict is of utmost importance, both practically and theoretically. Everyone encounters conflicts in everyday life. Conflict analysis and resolution play an important role in business, governmental, political and lawsuit disputes, labor-management negotiations, military operations and so on. Many mathematical models for conflict situations have been proposed and investigated [52]. Recently, rough set theory has been applied to analyze conflict situations. For example, the Pawlak conflict analysis model based on rough set has been proposed by many scholars [43,44,53]. The Pawlak conflict analysis model has proven to be an effective practical method in practice. However, some authors [12] have pointed out limitations of the Pawlak conflict analysis model: (1) What are the intrinsic reasons for the conflict? (2) How can a feasible consensus strategy be found? (3) Is it possible to satisfy all the agents? Other authors have discussed these three basic problems of the Pawlak conflict analysis model, with some important results. In this paper, we try to answer the second and third questions by a new method based on the dominance-based rough set approach proposed by Greco et al. [22–26].

As is well known, the optimal strategy which satisfies all agents in a conflict situation, i.e., the feasible consensus strategy, usually does not exist because there are different benefits and opinions. So, a sub-optimal feasible consensus strategy which satisfies the agents as much as possible is a reasonable goal. Along with this motivation, we consider the multiple agents conflict decision making problem as a kind of multiple attribute and multiple decision making problem and present a multi-decision preference dominance-based rough set model and a multi-decision $k$–grade preference dominance-based rough set model. We also present a general approach to solve the multi-agent conflict analysis decision making problem by using the proposed rough set models.

The dominance-based rough set approach was first established by Greco et al. [22–26] in order to deal with the inconsistencies in various types of multiple criteria decision analysis using rough set methodology. The basic idea behind the dominance-based rough set approach is to replace the equivalence relation in the Pawlak rough set [42] with the dominance relation, which permits taking into account the preference order in the value set of the criteria. The dominance-based rough set approach has attracted a great deal of attention and has been used for many valuable studies. Many extended models of the dominance-based rough set approach have been proposed, such as the graded dominance interval-valued rough set model [28,68], variable-precision dominance-based rough set approach [31], dominance-based rough set model in intuitionistic fuzzy information systems [27], variable-consistency dominance-based rough set approach [2,29], stochastic dominance-based rough set model [32], etc. Furthermore, several improved dominance-based rough set approaches have applied other mathematical theories to multiple criteria decision analysis [1,36,37,46,47,55,58,75]. Subsequently, Greco et al. [25] applied the dominance-based rough set approach to group decision making theory. They consider the decision of multiple decision makers by introducing specific concepts related to dominance with respect to minimal profiles of evaluations given by multiple decision makers. Their extension also provides a general methodology for rough approximations of partial preorders. Recently, Chakhar and Saad [9] study multicriteria classification problems in group decision
making and then propose a new methodology to support groups in multicriteria classification decision-making based on the dominance-based rough set approach [25]. Boggia et al. [6] developed a decision support system base for the dominance-based rough set approach, to assess the level of rural sustainable development in the region of Umbria, Italy. Fan et al. [20] present an approach for the decision analysis of a preference-ordered uncertain or possibilistic data table by combining the dominance-based rough set approach and the classical fuzzy rough set theory. James [30] applies the variable consistency dominance-based rough set approach to formulate airline service strategies by generating airline service decision rules that model passenger preferences for airline service quality. Greco et al. [26] propose to apply the Dominance-based Rough Set Approach (DRSA) to the results of multiple criteria decision aiding (MCDA) methods, in order to explain their recommendations in terms of rules involving conditions for evaluation criteria. At the same time, the multi-decision with multiple criteria or multiple attribute decision-making problem also is discussed by using the approach of the multiple attribute group decision or other methodology such as Data Envelopment Analysis, Analytic Hierarchy Process and TOPSIS, etc. [7,48,50,51].

The decision problem we consider, like the example of the Middle East conflict [43], is a problem of multiple criteria and multi-decision with preference group decision making. Classically, it is an extension of the multiple criteria decision analysis problems such as multiple criteria classification with imprecise evaluations and assignments [12], multi-attribute dominance fuzzy decision problems, multiple attribute dominance stochastic evaluation problems, multiple criteria group decision and so on [71,72]. Therefore, this paper transforms the multi-agent conflict decision making problem into a multi-decision with preference group decision making problem or a multiple attribute group decision making problem and then gives a new approach to solve it by establishing and using a multi-decision preference dominance-based rough set model. It also presents a new perspective and model for classical conflict decision making problems.

Inspired by the idea of a dominance-based rough set approach to group decision making theory given by Greco et al. [25], as well as the study of the work by Chakhar and Saad [9], our objective is to consider a kind of multiple attribute (criteria) choice or ranking problem with multiple decisions (or the multiple criteria group decision problem), in which the decision attribute has a different preference description for each action, as in the Middle East example. For this kind of multiple attribute choice problem with multiple decision agents, the original definition of Pawlak rough set, which involves the relation of indiscernibility to identify granules of objects used to build lower and upper approximations, could not handle the evaluation of every action according to the multiple decision preference. Based on the dominance-based rough set approach [53], with two dominance relations, we define one on the conditional attribute and the other on the decision attribute. Then, we present the lower and upper approximations of the dominance classes determined by the multiple decision attribute of each action using the dominance classes determined by the multiple attribute. Finally, we present the optimal decision according to the decision rule of the original rough set theory. That is, an optimal decision is established according to the lower approximation of the dominance classes for every action. In particular, multiple attribute and multiple decision with preference (MA&MD) decision making will be degenerated into the existing multiple criteria decision-making problem when there is only one decision attribute. Therefore, multiple attribute and multiple decision with preference (MA&MD) decision making is an extension of the existing work and also a generalization of the rough set theory.

The article is organized as follows. Section 2 contains the problem statement and basic definition of the MA&MD decision making problem of multiple attributes and multiple decision. Section 3 presents the multiple decision preference dominance-based rough set model, showing its generalized model as well as the multiple attribute and multiple decision making approach with preferred decision making. Meanwhile, a detailed discussion of the properties of the proposed models is included. In Section 4, we present a variable precision generalization of the multiple decision preference dominance-based rough set model established in Section 3. Also, several properties of this model are discussed in detail and the decision making method based on the variable precision multiple decision preference dominance-based rough set model is given. In Section 5, we present an application of the multi-decision preference dominance-based rough set approach to the conflict analysis decision making problem. Finally, we draw conclusions and set further research directions in Section 6.

2. Problem statement

The multiple decision with preference problem studied in this paper is essentially the multiple criteria group decision problem or other methodology such as Data Envelopment Analysis, Analytic Hierarchy Process and TOPSIS, etc. [7,48,50,51].

The decision problem may be conceived as a problem \( S = (A, C, D, E) \), where the sets Action, Conditional attribute, Decision attribute, and Evaluation (or domain of the conditional and decision attribute), respectively, are defined as follows: \( A \) is a finite set of actions \( a_i \), \( i = 1, 2, \ldots, n \); \( C \) is a finite set of conditional attributes \( C_j \), \( j = 1, 2, \ldots, |C| \); \( D \) is a finite set of decision attributes \( D_k \), \( k = 1, 2, \ldots, |D| \) (where \( | \cdot | \) denotes cardinality) and \( E \) is a finite set of the domain for the information function \( f(a_i, C_j) \) and \( g(a_i, D_k) \). In this article, the values of both information functions \( f(a_i, C_j) \) and \( g(a_i, D_k) \) are integers.

The value of \( f(a_i, C_j) \) describes the evaluation of action \( a_i \) based on criterion \( C_j \), and the value of \( g(a_i, D_k) \) describes the evaluation of action \( a_i \) by the decision-maker \( D_k \). That is, it describes the preference of decision-maker \( D_k \) for action \( a_i \). Broadly speaking, the greater the value of \( g(a_i, D_k) \), the greater the preference for action \( a_i \) by decision-maker \( D_k \).

We call \( S = (A, C, D, E) \) the MA&MD information system to avoid confusion.
In order to show the decision problem clearly, an example of a conflict situation for labor-management negotiations is presented in Table 1. There are five disputes or issues (conditional attributes) and four agents (decision attributes) with twelve feasible actions. The issues may be the employees’ incomes, working conditions, factory profits, and so on. The values of the integer are defined as follows: 0 – small (or bad), 1 – medium (or average), 2 – high (or good), 3 – highest (or excellent).

So, Table 1 describes a multiple attribute and multiple decision with preference decision information system for making a decision about the labor-management negotiations problem.

### 3. Multi-decision preference dominance-based approach to MA&MD problem

First, we recall the dominance principle, dominance relation, and dominance classes [22,23].

The basis of the dominance relation is the dominance principle: if action \( a_l \) dominates action \( a_r \), then \( a_r \) should be assigned to a class not worse than that to which \( a_l \) is assigned. Based on this dominance principle, Greco et al. [22] give the following definition of dominance relations for multiple criteria classifications.

Let \( P_q \) be an outranking relation on universe \( A \) with reference to criteria \( q \in C \) such that \( a_lP_qa_r \) means “\( a_l \) is at least as good as \( a_r \) with respect to criteria \( q \)” Suppose that \( P_q \) is a complete preorder, i.e., a strongly complete and transitive binary relation.

By the above definition of dominance relation, we present the dominance relation for our study problem as follows.

We define \( a_l \succ a_r \) by \( f(a_l, q) \succeq f(a_r, q) \) according to increasing preference, where \( q \in C \) and \( a_l, a_r \in A \). For any subset of conditional attribute \( Q \subseteq C \), \( a_l \succ a_r \) means that \( a_l \succ a_r \) for any \( q \in Q \); that is, \( a_l \) dominates \( a_r \) with respect to all attributes in \( Q \). Because the intersection of complete preorders is a partial preorder and \( P_Q \) also is a complete preorder for each \( q \in Q \subseteq C \) and \( P_Q = \bigcap_{q \in Q} P_q \), the dominance relation \( P_Q \) is a partial preorder. Given \( Q \subseteq C \) and \( a_l, a_r \in A \), let

\[
\begin{align*}
P_Q(a_l) &= \{ a_j \in A | a_lP_Qa_j \}, \\
P_Q(a_l) &= \{ a_j \in A | a_lP_Qa_j \} = \{ a_j \in A | f(a_j, Q) \leq f(a_l, Q) \},
\end{align*}
\]

represent the \( Q \)-dominating set and \( Q \)-dominated set with respect to \( a_l \) over the conditional attribute \( C \) of \( S = (A, C, D, E) \), respectively.

It can be easily seen that the binary relation \( P_C = \bigcap_{Q \subseteq C} P_Q \) is a dominance relation defined by the conditional attribute \( C \).

**Remark 3.1.** If the domain of decision attribute \( D \) does not have integer values, e.g. it has linguistic values or symbolic values, then \( P_Q(a_l) = \{ a_j \in A | a_lP_Qa_j \} = \{ a_j \in A | f(a_j, Q) \neq f(a_l, Q) \} \).

Similarly, we can also define the dominance relation over the decision attribute for the MA&MD information system \( S = (A, C, D, E) \) as follows. For any \( a_l, a_r \in A \),

\[
\begin{align*}
P_D(a_l) &= \{ a_j \in A | a_lP_Da_j \}, \\
P_D(a_l) &= \{ a_j \in A | a_lP_Da_j \} = \{ a_j \in A | g(a_j, D) \geq g(a_l, D) \},
\end{align*}
\]

represent the \( D \)-dominating and \( D \)-dominated sets with respect to \( a_l \) over the decision attribute set \( D \), respectively.

Actually, the \( D \)-dominating and \( D \)-dominated sets with respect to \( a_l \) also can be interpreted as the collective preference relation in the environment of group decision making.

Meanwhile, we call \( P_D \) the multiple decision preference dominance relation on \( S = (A, C, D, E) \).

In the following, we give the lower and upper approximations of the \( D \)-dominating set about the \( C \)-dominating set over the MA&MD information system \( S = (A, C, D, E) \).

### Table 1

Multi-criteria & multi-decision information system.

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Definition 3.1. Let $S = (A, C, D, E)$ be a MA&MD information system. For each $a_i \in A$, $P_C^+(a_i)$ and $P_D^+(a_i)$, respectively, are the $C$-dominating set and $D$-dominating set with respect to $a_i$ over $S = (A, C, D, E)$. Then, the lower and upper approximations of $P_D^+(a_i)$ about the dominance relation $P_C^+$ are defined as follows:

$$R(P_D^+(a_i)) = \{a_j | P_C^+(a_j) \subseteq P_D^+(a_i), a_j \in A\},$$

$$\overline{R}(P_D^+(a_i)) = \{a_j | P_C^+(a_j) \cap P_D^+(a_i) \neq \emptyset, a_j \in A\}.$$

We call $pos(P_D^+(a_i)) = R(P_D^+(a_i))$ the positive region of action $a_i$, $neg(P_D^+(a_i)) = A - \overline{R}(P_D^+(a_i))$ the negative region of action $a_i$ and $bn(P_D^+(a_i)) = \overline{R}(P_D^+(a_i)) - R(P_D^+(a_i))$ the boundary region of action $a_i$.

The lower approximation $R(P_D^+(a_i))$ includes all actions in the dominance classes $P_D^+(a_i)$ of action $a_i$ and the upper approximation $\overline{R}(P_D^+(a_i))$ includes all actions possibly included in the dominance classes $P_D^+(a_i)$ of action $a_i$. We can present the interpretation for the positive region, negative region and boundary region in the same way.

From the point of view of decision making, the lower approximation $R(P_D^+(a_i))$ can be regarded as all alternatives that dominate $a_i$ under the conditional attribute set $C$ with respect to the collective preference relation $P_D^+(a_i)$. Then the upper approximation $\overline{R}(P_D^+(a_i))$ can be regarded as all possible alternatives that dominated $a_i$ under the condition attribute set $C$ with respect to the collective preference relation $P_D^+(a_i)$. Furthermore, the following results are clear.

Remark 3.2. Let $S = (A, C, D, E)$ be a MA&MD information system. If $|D| = 1$, i.e., there is only one decision-maker, and $P_D^+(a_i) = \{a_j \in A | a_j \geq g(a_i, D)\}$; then the multi-decision preference dominance-based rough set model in Definition 3.1 degenerates into the dominance-based rough set approach [22–24].

Based on the above definition, for any $a_i, a_j \in A$, the following properties are clear.

**Property 1.**

$$R(P_D^+(a_i)) \subseteq P_D^+(a_i) \subseteq \overline{R}(P_D^+(a_i)).$$

**Property 2.**

$$R(P_D^+(a_i) \cap P_D^+(a_j)) = R(P_D^+(a_i)) \cap R(P_D^+(a_j)).$$

$$R(P_D^+(a_i) \cup P_D^+(a_j)) = R(P_D^+(a_i)) \cup R(P_D^+(a_j)).$$

**Property 3.**

$$R(P_D^+(a_i) \cup P_D^+(a_j)) \supseteq R(P_D^+(a_i)) \cup R(P_D^+(a_j)).$$

$$R(P_D^+(a_i) \cap P_D^+(a_j)) \subseteq R(P_D^+(a_i)) \cap R(P_D^+(a_j)).$$

From the above definition, we present the lower and upper approximations for the action set $A$ with respect to the multiple decision preference dominance relation $P_D^+$ as follows:

$$R(P_D^+(A)) = \bigcap_{a_i \in A} R(P_D^+(a_i)) = \bigcap_{a_i \in A} \{a_j | P_C^+(a_j) \subseteq P_D^+(a_i), a_j \in A\},$$

$$\overline{R}(P_D^+(A)) = \bigcap_{a_i \in A} \overline{R}(P_D^+(a_i)) = \bigcap_{a_i \in A} \{a_j | P_C^+(a_j) \cap P_D^+(a_i) \neq \emptyset, a_j \in A\}.$$

We call $pos(P_D^+(A)) = R(P_D^+(A))$ the positive region, $neg(P_D^+(A)) = A - \overline{R}(P_D^+(A))$ the negative region and $bn(P_D^+(A)) = \overline{R}(P_D^+(A)) - R(P_D^+(A))$ the boundary region of $A$ with multiple decision preference dominance relation $P_D^+$, respectively.

It is easy to give the above interpretation of the lower and upper approximations for action set $A$ with the multiple decision preference dominance relation $P_D^+$.

By this definition, the following property is available.

**Property 4.**

$$R(P_D^+(A)) \subseteq A, \quad \overline{R}(P_D^+(A)) \subseteq A.$$

Remark 3.3. In general, it can be easily verified that the relation $R(P_D^+(A)) \subseteq \overline{R}(P_D^+(A))$ does not hold.

Based on the lower approximation definition of the $D$-dominating set $P_D^+(A)$ of action set $A$ for MA&MD information system $S = (A, C, D, E)$, we give the optimal decision alternative for MA&MD information system $S = (A, C, D, E)$ as follows:
(1) If there exists \( a_i \in \mathcal{R}(P_D^+(A)) \), then \( a_i \in A \) is the optimal decision alternative of the MA&MD information system \( S = (A, C, D, E) \).

The above decision making shows that all alternatives included in the lower approximation of action set \( A \) must be the optimal decision alternative for MA&MD information system \( S = (A, C, D, E) \), in which all agents will agree to the action. Meanwhile, we know from the decision making that there is no optimal decision alternative satisfying all agents when the lower approximation \( \mathcal{R}(P_D^+(A)) \) is an empty set.

To illustrate the principle of the rough set approach to modeling multiple decisions with preference for multiple criteria decision analysis, we consider the example of the conflict situation for labor-management negotiations in Section 2. As for the classical Pawlak approach to conflict analysis, it transforms the conflict analysis into Boolean reasoning and then presents a rough set-based approach to solve it [31]. Here we transform the conflict-resolving problem into a decision making problem of multiple attributes and multi-decision with preference and then use the multiple decision preference dominance-based rough set approach to solve it.

**Example 3.1.** In the labor-management negotiations conflict situation, there are five disputes (issues) and four agents (players). Suppose that there are twelve feasible alternatives pre-established by experts or obtained by other approaches and that every agent has given its preference in advance for every feasible action. The results are presented in Table 1.

We can easily obtain the following result by using the multiple decision preference dominance-based rough set model.

\[
\mathcal{R}(P_D^+(A)) = \bigcap_{a_i \in A} \mathcal{R}(P_D^+(a_i)) = \bigcap_{a_i \in A} \{a_i | P_C^+(a_i) \subseteq P_D^+(a_i)\} = \emptyset.
\]

By the decision principle (1), we know that there is no optimal alternative for the labor-management conflict \( S = (A, C, D, E) \) since \( \mathcal{R}(P_D^+(A)) = \emptyset \).

So, the optimal decision alternative may not exist in practice for a multiple attribute choice problem with multiple decision-makers. Like the example of the Middle East, the optimal decision satisfying all agents may not exist.

Similarly, it could be an empty set of the lower approximation of action set \( A \) for the MA&MD information system. That is, the optimal decision alternative satisfying all decision-makers could not exist for a given MA&MD information system. This is not the expected outcome for every decision-maker in practice. In view of this problem, we give an improved version of the multiple decision preference dominance-based rough set model and then adapt it to the MA&MD decision making problem in reality. That is, this is the multiple decision \( k \)-grade preference dominance-based rough set model.

We first define a \( k \)-grade dominance relation over the decision attribute set and then present the \( k \)-grade lower and upper approximations of the dominance classes for the decision attribute with every action by using the dominance classes of the conditional attribute. Thus, a sub-optimal decision making alternative is given by the \( k \)-grade lower approximation. This is a feasible alternative with \( k \) decision makers reaching consensus for the MA&MD decision problem.

First, we present the definition of the multi-decision \( k \)-grade preference dominance relation.

**Definition 3.2.** Let \( S = (A, C, D, E) \) be a MA&MD information system. \( d_i \in K \subseteq D \). We call

\[
P_D^{k+}(a_i) = \{a_i \in A | g(a_i, d_i) \geq g(a_i, d_i), 0 \leq k \leq |K|, |K| \leq |D|\},
\]

\[
P_D^{k-}(a_i) = \{a_i \in A | g(a_i, d_i) < g(a_i, d_i), 0 \leq k \leq |K|, |K| \leq |D|\},
\]

the \( k \)-grade \( D \)-dominating set and \( k \)-grade \( D \)-dominated set with respect to \( a_i \) over decision attribute \( D \), respectively.

Meanwhile, we call \( P_D^{k\pm} \) the multiple decision \( k \)-grade preference dominance relation over decision attribute \( D \).

From this definition, we know that the \( k \)-grade \( D \)-dominating set of action \( a_i \) includes the action that dominates \( a_i \) at least, the \( k \) decision attributes within \( D \). The \( k \)-grade \( D \)-dominated set has a similar interpretation.

By the definition, the following results are available.

**Remark 3.4.** If \( k = |D| \), then \( P_D^{k\pm}(a_i) = P_D^+(a_i) \) for \( a_i \in A \).

**Remark 3.5.** If \( k = 0 \), i.e., \( K = \emptyset \), then the following relation holds.

\[
P_D^{k\pm}(a_i) = \{a_i \in A | g(a_i, d_i) < g(a_i, d_i), a_i \in A\} = P_D^-(a_i).
\]

So, the \( k \)-grade \( D \)-dominating set \( P_D^{k\pm}(a_i) \) degenerates the \( D \)-dominated set over the decision attribute \( D \).

It can be easily seen that the requirement of the multiple decision preference dominance relation in Definition 3.1 is more rigorous than the multiple decision \( k \)-grade preference dominance relation in Definition 3.2. It demands the relation \( g(a_i, d_i) \geq g(a_i, d_i) \) hold with every decision attribute \( d_i \) in \( D \) for every \( a_i, a_j \in A \) in the multiple decision preference dominance relation. However, at least \( k \) decision attributes \( d_i \) in \( D \) must satisfy the relation \( g(a_i, d_i) \geq g(a_i, d_i) \) for every \( a_i, a_j \in A \). Therefore, the following relationship is inferred.

**Property 5.** For any \( a_i \in A, 1 \leq k \leq |K|, K \subseteq D \) the following relations hold:
(1) \( P_D^+(a_i) \subseteq P_D^{k_+}(a_i) \),
(2) \( P_D^-(a_i) \subseteq P_D^{k_-}(a_i) \).

Based on the multiple decision \( k \)-grade preference dominance relation over decision attribute \( D \) of \( S = (A, C, D, E) \), we present the lower and upper approximations of the \( k \)-grade \( D \)-dominating set about the \( C \)-dominating set for the MA&MD information system as follows.

**Definition 3.3.** Let \( S = (A, C, D, E) \) be a MA&MD information system. For each \( a_i \in A \), the sets \( P_D^+(a_i) \) and \( P_D^{k_+}(a_i) \), respectively, are the \( C \)-dominating set and \( k \)-grade \( D \)-dominating set with respect to \( a_i \) over \( S = (A, C, D, E) \). Then the lower and upper approximations of \( P_D^{k_+}(a_i) \) about the dominance relation \( P_C^+ \) are defined as follows:

\[
\begin{align*}
R(P_D^{k_+}(a_i)) &= \{ a_j | P_C^+(a_j) \subseteq P_D^{k_+}(a_i), a_j \in A \}, \\
\overline{R}(P_D^{k_+}(a_i)) &= \{ a_j | P_C^+(a_j) \cap P_D^{k_+}(a_i) \neq \emptyset, a_j \in A \}.
\end{align*}
\]

We call \( pos(P_D^{k_+}(a_i)) = R(P_D^{k_+}(a_i)) \) the positive region of action \( a_i \), \( neg(P_D^{k_+}(a_i)) = A - \overline{R}(P_D^{k_+}(a_i)) \) the negative region of action \( a_i \), and \( bn(P_D^{k_+}(a_i)) = R(P_D^{k_+}(a_i)) - R(P_D^{k_+}(a_i)) \) the boundary region of action \( a_i \).

We can interpret the meanings of the lower and upper approximations of \( P_D^{k_+}(a_i), a_i \in A \) similarly to the lower and upper approximations in **Definition 3.1.** At the same time, the following relationships are apparent.

**Property 6.** For any \( a_i \in A, 1 \leq k \leq |K|, K \subseteq D \), the following relations hold:

(1) \( R(P_D^+(a_i)) \subseteq R(P_D^{k_+}(a_i)) \),
(2) \( \overline{R}(P_D^+(a_i)) \subseteq \overline{R}(P_D^{k_+}(a_i)) \).

By **Definition 3.3**, we present the lower and upper approximations for the \( k \)-grade \( D \)-dominating set \( P_D^{k_+}(A) \) of action set \( A \) about the dominance relation \( P_C^+ \) as follows:

\[
\begin{align*}
R(P_D^{k_+}(A)) &= \bigcap_{a_i \in A} R(P_D^{k_+}(a_i)) = \bigcap_{a_i \in A} \{ a_j | P_C^+(a_j) \subseteq P_D^{k_+}(a_i) \}, \\
\overline{R}(P_D^{k_+}(A)) &= \bigcap_{a_i \in A} \overline{R}(P_D^{k_+}(a_i)) = \bigcap_{a_i \in A} \{ a_j | P_C^+(a_j) \cap P_D^{k_+}(a_i) \neq \emptyset \}.
\end{align*}
\]

We call \( pos(P_D^{k_+}(A)) = R(P_D^{k_+}(A)) \) the positive region, \( neg(P_D^{k_+}(A)) = A - \overline{R}(P_D^{k_+}(A)) \) the negative region and \( bn(P_D^{k_+}(A)) = R(P_D^{k_+}(A)) - R(P_D^{k_+}(A)) \) the boundary region of \( A \) with the multiple decision \( k \)-grade preference dominance relation \( P_D^{k_+} \).

Similarly, we have the following properties for these lower and upper approximations.

**Property 7.** For any \( 1 \leq k \leq |K|, K \subseteq D \), the following relations hold:

(1) \( R(P_D^{k_+}(A)) \subseteq A \),
(2) \( \overline{R}(P_D^{k_+}(A)) \subseteq A \),
(3) \( R(P_D^+(A)) \subseteq R(P_D^{k_+}(A)) \),
(4) \( \overline{R}(P_D^+(A)) \subseteq \overline{R}(P_D^{k_+}(A)) \).

It is readily known that the lower and upper approximations with \( k \)-grade preference dominance relation \( P_D^{k_+} \) have properties similar to Properties 1 through 3.

With the lower approximations of the \( k \)-grade \( D \)-dominating set \( P_D^{k_+}(A) \) of action set \( A \) over \( S = (A, C, D, E) \), we present the optimal decision alternative as follows:

(1) If there exists \( a_i \in R(P_D^{k_+}(A)) \), then \( a_i \) is the \( k \)-grade optimal decision alternative of MA&MD information system \( S = (A, C, D, E) \).

This decision alternative shows that at least \( k \) decision-makers will approve the alternative \( a_i \) if \( a_i \in R(P_D^{k_+}(A)) \neq \emptyset \). That is, \( a_i \) is a feasible consensus alternative with the support of at least \( k \) decision makers for the MA&MD information system \( S = (A, C, D, E) \).

Because there may not exist an optimal decision alternative for a MA&MD information system, we propose the multiple decision with \( k \)-grade preference dominance-based rough set model by introducing the concept of a \( k \)-grade preference dominance relation over decision attribute \( D \) of \( S = (A, C, D, E) \). Then, there must exist an optimal decision alternative that
satisfies at least \( k \) decision makers. However, we wish to find an optimal decision alternative approved by as many decision-makers as possible when an optimal decision alternative approved by all decision-makers of the MA&MD information system \( S = (A, C, D, E) \) does not exist. Therefore, the key problem is how to find the maximum value \( k \) that satisfies the condition \( R(P_D^{k+1}(A)) \neq \emptyset \) for MA&MD information system \( S = (A, C, D, E) \).

In the following paragraphs, we present the concept of a maximum \( k \)-grade optimal decision alternative for MA&MD information system \( S = (A, C, D, E) \).

**Definition 3.4.** Let \( S = (A, C, D, E) \) be a MA&MD information system. If there exists \( k \in [1, |D|] \) that satisfies the following two conditions:

1. \( R(P_D^{k+1}(A)) \neq \emptyset \), and
2. \( R(P_D^{k+1}(A)) = \emptyset \).

then, for any \( a_i \in R(P_D^k(A)) \), we call \( a_i \) the maximum \( k \)-grade optimal decision alternative of MA&MD information system \( S = (A, C, D, E) \).

**Definition 3.4** also gives an approach to find the maximum \( k \)-grade optimal decision alternative for MA&MD information system \( S = (A, C, D, E) \).

In general, let \( k = |D| \), if \( R(P_D^k(A)) \neq \emptyset \). Then there exists a feasible consensus decision alternative with the approval of at least \( k \) decision-makers. Otherwise, let \( k = |D| - 1 \), if \( R(P_D^k(A)) \neq \emptyset \). Then we find the maximum \( k \)-grade optimal decision alternative for MA&MD information system \( S = (A, C, D, E) \). We repeat this process several times to find \( k \in [1, |D|] \) and satisfy \( R(P_D^k(A)) \neq \emptyset \). Meanwhile, if \( k = 0 \) and \( R(P_D^k(A)) = \emptyset \), then there is not a maximum \( k \)-grade optimal decision alternative for MA&MD information system \( S = (A, C, D, E) \).

In the following, we present the algorithm to find the maximum \( k \)-grade optimal decision alternative for MA&MD information system \( S = (A, C, D, E) \).

**Algorithm 1. Algorithm for maximum \( k \)-grade optimal decision alternative of the MA&MD information system**

1: Initialize the value of \( k = |D| \);
2: Calculate \( R(P_D^{k+1}(A)) \);
3: If \( R(P_D^{k+1}(A)) \neq \emptyset \);
   go to 4.
   Otherwise,
   \( k = k - 1 \);
   go back to 3.
4: Return maximum value \( k \).

In the following, an example is presented to show the basic idea and verify the conclusions in this section.

**Example 3.2** (Continued from Example 3.1). Let \( k = 3 \). Then the lower approximation of 3-grade \( D \)-dominating set \( P_D^{3+}(A) \) of action set \( A \) for \( S = (A, C, D, E) \) is as follows:

\[
R(P_D^{3+}(A)) = \bigcap_{a_i \in A} R(P_D^{3+}(a_i)) = \bigcap_{a_i \in A} \{a_i| P_C(a_i) \subseteq P_D(a_i)\} = \{a_7\}.
\]

By the optimal decision making (II), we know that action \( a_7 \) is the optimal decision alternative with a three-grade preference dominance relation \( P_D^{3+} \) for MA&MD information system \( S = (A, C, D, E) \). That is, action \( a_7 \) is a feasible consensus decision for solving the labor-management conflict. Thus, we found the feasible consensus strategy for the labor-management negotiation conflict situation by using the proposed method.

Furthermore, it can be easily seen that \( a_7 \) also is the maximum 3-grade optimal decision alternative for MA&MD information system \( S = (A, C, D, E) \) by combining the results with Example 3.1.

In a recent study, Bi and Chen [3] consider a group sorting decision problem with an extended dominance-based rough set. The background of the decision problem studied by Bi and Chen is similar to the multiple decision dominance with preference problem considered in this paper. Bi and Chen introduce a specific concept, minimal profiles, and estimate the group’s preference by calculating lower approximates of minimal profiles given the consistent condition. They obtain a kind of decision rule with the form “If ..., then ...” In fact, there may exist some inconsistency with two different decision rules for the same decision, but the descriptions in the conditional part could not be identical. Moreover, a large amount of calculation may be required to determine the group’s preferences.
4. Variable precision multi-decision preference dominance-based approach to MA&MD problem

Generally speaking, the multiple decision preference dominance-based rough set model given in Section 3 is a qualitative model that defines the lower and upper approximations by the inclusion relationship between the approximated objects and the dominance classes according to the two dominance relations over the conditional attribute and decision attribute. However, the model does not consider the degree of overlap between the dominance classes over the conditional attribute and the decision attribute. This motivates us to develop a quantitative approach to the multiple decision preference dominance-based rough set. Therefore, we give a generalization of the multiple decision preference dominance-based rough set model proposed in Section 3; that is, the variable precision multiple decision preference dominance-based rough set models. Moreover, we also incorporate the generalized model into the decision method.

First of all, we give the concept of inclusion degree.

**Definition 4.1.** [67,70] Let \((L, \preceq)\) be a partial ordered set. For any \(a, b \in L\), there is a number \(\pi(b/a)\) that satisfies the following conditions:

1. \(0 \leq \pi(b/a) \leq 1\);
2. If \(a \preceq b\), then \(\pi(b/a) = 1\);
3. If \(a \preceq b \preceq c\), then \(\pi(a/c) \leq \pi(a/b)\);
4. If \(a \preceq b\) and for any \(c \in L\), \(\pi(a/c) \leq \pi(b/c)\) holds.

Then we call \(\pi(\bullet)\) an inclusion degree on \(L\).

In this definition, (1) shows the normality of the inclusion degree, (2) describes the consistency of the inclusion degree with the classical including relation, and (3) and (4) express the monotonicity of the inclusion degree.

**Remark 4.1.** The inclusion degree defined on a partially ordered set given in **Definition 4.1** is a natural generalization of the crisp set. In Refs. [4,5], Bustince et al. present a concept of DI-subsethood measure for fuzzy sets as follows:

If \(\sigma_{DI} : F(X) \times F(X) \to [0, 1] \) (where \(F(X)\) stands for all fuzzy subsets of universe \(X\)) satisfies:

(a) \(\sigma_{DI}(A, B) = 1\) if and only if \(A \preceq B\) as defined by Zadeh; that is, \(\mu_A(x_i) \leq \mu_B(x_i)\) for all \(i \in \{1, 2, \ldots, n\}\),
(b) \(\sigma_{DI}(A, A^c) = 0\) (where \(A^c\) stands for the complement of fuzzy set \(A\)) if and only if \(\mu_A(x_i) = 1\) for all \(i \in \{1, 2, \ldots, n\}\),
(c) If \(A \preceq B\), then \(\sigma_{DI}(A, C) \geq \sigma_{DI}(B, C)\) and \(\sigma_{DI}(C, A) \leq \sigma_{DI}(C, B)\),

then we say \(\sigma_{DI}\) is a fuzzy DI-subsethood measure on \(X\).

It can be easily seen that the inclusion degree \(\pi(\bullet)\) on the partial ordered set \(L\) will be the DI-subsethood measure given by Bustince et al. [4,5] when we define the partial ordered relation \(\preceq\) as the fuzzy inclusion relation given by Zadeh. So, the inclusion degree in **Definition 4.1** can also be regarded as a kind of DI-subsethood measure.

In the following, we give the lower and upper approximations of variable precision multiple decision preference dominance-based rough set by using the concept of inclusion degree.

**Definition 4.2.** Let \(S = (A, C, D, E)\) be a MA&MD information system. For each \(a_i \in A\), \(P^c_\beta(a_i)\) and \(P^\beta_\beta(a_i)\), respectively, are the \(C\)-dominating set and \(D\)-dominating set with respect to \(a_i\) over \(S = (A, C, D, E)\). Let \(0 \leq \beta < 0.5\). The lower and upper approximations of \(P^\beta_\beta(a_i)\) about the dominance relation \(P^c_\beta\) with respect to parameter \(\beta\) are defined as follows:

\[
\begin{align*}
\mathcal{R}_\beta(P^\beta_\beta(a_i)) &= \{a_j \in A | \pi(P^\beta_\beta(a_i)/P^c_\beta(a_j)) \geq 1 - \beta\}, \\
\mathcal{L}_\beta(P^\beta_\beta(a_i)) &= \{a_j \in A | \pi(P^\beta_\beta(a_i)/P^c_\beta(a_j)) > \beta\}.
\end{align*}
\]

Moreover, the \(\beta\)-boundary region and \(\beta\)-negative region of \(P^\beta_\beta(a_i)\) are defined as follows:

\[
\begin{align*}
bn_\beta(P^\beta_\beta(a_i)) &= \{a_j \in A | \beta < \pi(P^\beta_\beta(a_i)/P^c_\beta(a_j)) < 1 - \beta\}, \\
\neg_\beta(P^\beta_\beta(a_i)) &= \{a_j \in A | \pi(P^\beta_\beta(a_i)/P^c_\beta(a_j)) \leq \beta\}.
\end{align*}
\]

Meanwhile, some scholars give another form of the variable precision rough set model following the idea of Ziarko [69]: the asymmetric boundary region variable precision rough set. Here we also give this model’s lower and upper approximations in detail as follows:

**Definition 4.2.** Let \(S = (A, C, D, E)\) be a MA&MD information system. For each \(a_i \in A\), \(P^c_\beta(a_i)\) and \(P^\beta_\beta(a_i)\) respectively, are the \(C\)-dominating set and \(D\)-dominating set with respect to \(a_i\) over \(S = (A, C, D, E)\). Let \(0 \leq l < u < 1\). Then, the lower and upper approximations of \(P^\beta_\beta(a_i)\) about the dominance relation \(P^c_\beta\) with respect to parameters \(l\) and \(u\) are defined as follows:

\[
\begin{align*}
\mathcal{R}_l(P^\beta_\beta(a_i)) &= \{a_j \in A | \pi(P^\beta_\beta(a_i)/P^c_\beta(a_j)) \geq u\}, \\
\mathcal{L}_u(P^\beta_\beta(a_i)) &= \{a_j \in A | \pi(P^\beta_\beta(a_i)/P^c_\beta(a_j)) \geq l\}.
\end{align*}
\]
Similarly, the boundary region and negative region of $P^+_D(a_i)$ with respect to parameters $l$ and $u$ are as follows:

$$bn_{lu}(P^+_D(a_i)) = \{ a_i \in A | l < \pi(P^+_D(a_i)/P^+_C(a_i)) < u, a_i \in A \},$$

$$neg_{lu}(P^+_D(a_i)) = \{ a_i \in A | \pi(P^+_D(a_i)/P^+_C(a_i)) < l, a_i \in A \}.$$

At the same time, we present the lower and upper approximations for the action set $A$ with respect to multiple decision preference dominance relation $P^+_D$ about the parameter $\beta(0 < \beta < 0.5)$ as follows:

$$R_b(P^+_D(A)) = \bigcap_{a_i \in A} R_b(P^+_D(a_i)) = \bigcap_{a_i \in A} \{ a_i | \pi(P^+_D(a_i)/P^+_C(a_i)) \geq 1 - \beta, a_i \in A \},$$

$$R_\beta(P^+_D(A)) = \bigcap_{a_i \in A} R_\beta(P^+_D(a_i)) = \bigcap_{a_i \in A} \{ a_i | \pi(P^+_D(a_i)/P^+_C(a_i)) > \beta, a_i \in A \}. $$

Similarly, for any $0 < l < u < 1$, we have

$$R_l(P^+_D(A)) = \bigcap_{a_i \in A} R_l(P^+_D(a_i)) = \bigcap_{a_i \in A} \{ a_i | \pi(P^+_D(a_i)/P^+_C(a_i)) > u, a_i \in A \},$$

$$R_u(P^+_D(A)) = \bigcap_{a_i \in A} R_u(P^+_D(a_i)) = \bigcap_{a_i \in A} \{ a_i | \pi(P^+_D(a_i)/P^+_C(a_i)) \geq l, a_i \in A \}.$$

By the above definitions, the following results are available.

**Property 4.1.** Let $S = (A, C, D, E)$ be a MA&MD information system. For each $a_i, a_j \in A$ and $0 < \beta < 0.5$. There are

1. $R_b(P^+_D(a_i)) \subseteq A,$
2. $R_b(P^+_D(a_i)) \subseteq R_b(P^+_D(a_i));$
3. $R_b(P^+_D(a_i) \cap P^+_D(a_j)) \subseteq R_b(P^+_D(a_i)) \cap R_b(P^+_D(a_j));$
4. $R_b(P^+_D(a_i) \cup P^+_D(a_j)) \supseteq R_b(P^+_D(a_i)) \cup R_b(P^+_D(a_j));$
5. $R_b(P^+_D(a_i)) \subseteq A.$

It can be easily seen that there also are analogous results for the properties of $R_u(P^+_D(a_i))$ and $R_\beta(P^+_D(a_i)).$

In the following, we present the decision making process based on the variable precision multiple decision preference dominance-based rough set model.

(III) If there exists $a_i \in R_\beta(P^+_D(A))$, then $a_i \in A$ is the optimal decision alternative with the decision precision $1 - \beta$ of MA&MD information system $S = (A, C, D, E)$.

In fact, the precision parameter $\beta$ also can be understood in terms of the risk of decision making failure. That is, the risk of failure in decision making will be at most $\beta$ when the decision-maker takes the action $a_i \in R_\beta(P^+_D(A)).$

Next, we give the variable precision multiple decision preference $k$-grade dominance-based rough set model. Because both the ideas and the properties are similar to the variable precision multi-decision preference dominance-based rough set model, here we only present the definitions for this model.

**Definition 4.3.** Let $S = (A, C, D, E)$ be a MA&MD information system. For each $a_i \in A, P^+_C(a_i)$ and $P^+_D(a_i)$, respectively, are the $C$-dominating set and $k$-grade $D$-dominating set with respect to $a_i$ over $S = (A, C, D, E)$. Let $0 < \beta < 0.5$. The lower and upper approximations of $P^+_D(a_i)$ about the dominance relation $P^+_C$ with respect to parameter $\beta$ are defined as follows:

$$R_b(P^+_D(a_i)) = \{ a_i \in A | \pi(P^+_D(a_i)/P^+_C(a_i)) \geq 1 - \beta \},$$

$$R_\beta(P^+_D(a_i)) = \{ a_i \in A | \pi(P^+_D(a_i)/P^+_C(a_i)) > \beta \}.$$

Moreover, the $\beta$-boundary region and $\beta$-negative region of $P^+_D(a_i)$ are defined as follows:

$$bn_{\beta}(P^+_D(a_i)) = \{ a_i \in A | \beta < \pi(P^+_D(a_i)/P^+_C(a_i)) < 1 - \beta \},$$

$$neg_{\beta}(P^+_D(a_i)) = \{ a_i \in A | \pi(P^+_D(a_i)/P^+_C(a_i)) \leq \beta \},$$

**Definition 4.3.** Let $S = (A, C, D, E)$ be a MA&MD information system. For each $a_i \in A, P^+_C(a_i)$ and $P^+_D(a_i)$, respectively, are the $C$-dominating set and $k$-grade $D$-dominating set with respect to $a_i$ over $S = (A, C, D, E)$. Let $0 < l < u < 1$; then, the lower and upper approximations of $P^+_D(a_i)$ about the dominance relation $P^+_C$ with respect to parameters $l$ and $u$ are defined as follows:

$$R_b(P^+_D(a_i)) = \{ a_i \in A | \pi(P^+_D(a_i)/P^+_C(a_i)) \geq u \},$$

$$R_u(P^+_D(a_i)) = \{ a_i \in A | \pi(P^+_D(a_i)/P^+_C(a_i)) \geq l \}. $$
Moreover, the $\beta$-boundary region and $\beta$-negative region of $P^+_D(a_i)$ are defined as follows:

\[
\begin{align*}
\text{bm}_u(P^+_D(a_i)) &= \{a_j \in A | I < \pi(P^+_D(a_i)/P^+_C(a_j)) < u\}, \\
neg(P^+_D(a_i)) &= \{a_j \in A | \pi(P^+_D(a_i)/P^+_C(a_j)) \leq I\}.
\end{align*}
\]

Similar to the previous discussions, we can describe the decision making and its precision by using the variable precision multiple decision preference $k$-grade dominance-based rough set model. We also can give the maximum $k$-grade optimal decision choice with precision $\beta$ and its algorithm. Both of them correspond to the concepts defined in Section 3.

In the following, we present an example to show the basic idea and verify the conclusions presented in this section.

**Example 4.1 (Continued from Example 3.1).** Let $X, Y$ be two finite sets. We use the following formula to calculate the inclusion degree between $X$ and $Y$:

\[
\pi(X/Y) = \frac{|X \cap Y|}{|X|} (X \neq \emptyset).
\]

Let $\beta = 0.4$. Then we can obtain the lower approximation of $P^+_D(A)$ about the dominance relation $P^+_C$ with respect to parameter $\beta$ as follows:

\[
R_{0.4}(P^+_D(A)) = \bigcap_{a_i \in A} R_{0.4}(P^+_D(a_i)) = \bigcap_{a_i \in A} \{a_j | \pi(P^+_D(a_i)/P^+_C(a_j)) \geq 0.6\} = \{a_7\}.
\]

This result shows that action $a_7$ is the optimal decision alternative with decision precision 0.6.

It is easy to see that the same optimal decision making is obtained with the multiple decision preference $k$-grade dominance-based rough set model and the variable precision multiple decision preference dominance-based rough set model for the MA&Md information system $S = (A, C, D, E)$. However, it is a coincidence for this example. That is, there will be different optimal decision alternatives when we use different decision models.

5. Application to the multi-agent conflict analysis problem of multi-decision preference dominance rough set

In this section, we present an application of the multi-decision preference dominance rough set model given in Section 3. We apply the multi-decision preference dominance rough set approach to solve the multi-agent conflict analysis problem. Then we present a new method to handle the conflict decision making problem by using the generalized dominance-based rough set theory.

We reconsider the example of the conflict situation for the labor-management negotiations which was presented in Section 2.

In this conflict situation, there are five disputes (issues) and four agents (players), facing twelve feasible actions, and every agent has announced its preference for every feasible action.

According to the opinion proposed by Deja [14], the conflict analysis decision task is decomposed into three problems:

(a) What are the conflict reasons?
(b) How can the consensus be found?
(c) It is possible to satisfy all agents?

In fact, this kind of conflict decision making problem has been analyzed by transforming the conflict analysis problem and the conflict-resolving problem into a Boolean-reasoning problem [52] and a proposed approach to solving them. In this paper, we transform the conflict analysis problem and the conflict-resolving process into a decision making problem of multiple criteria and multi-decision with preference and focus on the problems (b) and (c). Then we use the dominance-based rough set approach with preference proposed in Sections 3 and 4 respectively, to solve these two problems.

By applying the multi-decision preference dominance-based rough set approach proposed in Section 3 and Section 4 to labor-management negotiations, we can easily obtain the following results:

First, we use the multi-decision preference dominance-based rough set method given in Section 3. The results are given as follows.

(i) Let $k = |D| = 5$. That is, we find the alternatives which satisfy all agents. Then we can calculate the lower approximation $P^+_D$ about $S = (A, C, D, E)$ as follows:

\[
\mathcal{R}(P^+_D(A)) = \bigcap_{a_i \in A} \mathcal{R}(P^+_D(a_i)) = \bigcap_{a_i \in A} \{a_j | P^+_C(a_i) \subseteq P^+_D(a_j)\} = \emptyset.
\]

By decision rule (I), we know that an optimal choice decision $S = (A, C, D, E)$ does not exist because $\mathcal{R}(P^+_D(A)) = \emptyset$. 
(ii) Let \( k = 4 \). That is, we find the alternatives which satisfy four agents out of all the agents in the conflict analysis situation. Then we can calculate the lower approximation \( P_D^{4+} \) about \( S = \{A, C, D, E\} \) as follows:

\[
R(P_D^{4+}(A)) = \bigcap_{a_i \in A} R(P_D^{4+}(a_i)) = \bigcap_{a_i \in A} \{a_i | P_C^4(a_i) \subseteq P_D^{4+}(a_i)\} = \emptyset.
\]

Similarly, we know that there is no sub-optimal choice decision alternative which satisfies four agents out of all the agents \( S = \{A, C, D, E\} \) because \( R(P_D^{4+}(A)) = \emptyset \) according to the decision rule (II).

(iii) Let \( k = 3 \). Then we can calculate the lower approximation \( P_D^{3+} \) about \( S = \{A, C, D, E\} \) as follows:

\[
R(P_D^{3+}(A)) = \bigcap_{a_i \in A} R(P_D^{3+}(a_i)) = \bigcap_{a_i \in A} \{a_i | P_C^3(a_i) \subseteq P_D^{3+}(a_i)\} = \{a_7\}.
\]

By decision rule (II), we know that action \( a_7 \) is the sub-optimal choice decision alternative with three-grade preference dominance relation \( P_D^{3+} \) for the conflict situation \( S = \{A, C, D, E\} \). That is, action \( a_7 \) is a consensus feasible decision for solving this conflict analysis problem, which is agreed on by three agents out of all the agents in the conflict situation \( S = \{A, C, D, E\} \).

The results show that there is no optimal alternative which satisfies all agents in the conflict situation \( S = \{A, C, D, E\} \). Moreover, there is a sub-optimal alternative which satisfies four agents in the conflict situation \( S = \{A, C, D, E\} \). However, there is no sub-optimal alternative which satisfies three agents in the conflict situation \( S = \{A, C, D, E\} \). That is, we find a feasible consensus strategy which satisfies as many agents as possible. So, we not only show whether there exists an alternative that satisfies all agents, but also present a method to find the consensus for a given conflict situation. Therefore, we answer the second and third questions proposed by Deja [13] for the classical Pawlak conflict analysis decision making model [43,44].

Next, we use the variable precision multi-decision preference dominance-based rough set method given in Section 4. The results are given as follows.

Let \( X, Y \) be two finite sets. We use the formula \( \pi(X/Y) = \frac{E(Y|X)}{E(X)}(X \neq \emptyset) \) to calculate the inclusion degree between \( X \) and \( Y \). Let \( \beta = 0.4 \). Then we can obtain the lower approximation of \( P_D^\beta(A) \) about \( P_C \) with respect to parameter \( \beta \) as follows:

\[
R_{0.4}(P_D^\beta(A)) = \bigcap_{a_i \in A} R_{0.4}(P_D^\beta(a_i)) = \bigcap_{a_i \in A} \{a_i | \pi(P_D^\beta(a_i)/P_C(a_i)) \geq 0.6\} = \{a_7\}.
\]

This result shows that the action \( a_7 \) is the optimal choice decision with decision precision 0.6. In fact, the decision precision \( \beta \) also can be regarded as the consensus level among the agents in a given conflict situation. So, we present another method to find a feasible consensus strategy which satisfies as many agents as possible for a given conflict situation.

As far as the results are concerned, we apply, respectively, the multi-decision preference dominance-based rough set model and the variable precision multi-decision preference dominance-based rough set model to the conflict situation \( S = \{A, C, D, E\} \) and obtain the same optimal decision. As above, it is just a coincidence for this example. That is, there will be different optimal decisions when we use different decision models and the above two decision models do not have alternatives in practice.

So far, we have presented an approach to a kind of multiple attribute and multiple decision making problem (i.e., a multiple attribute conflict analysis decision making problem) by using the multi-decision preference dominance-based rough set model. Compared to previous studies of dominance-based rough set and other improved rough set approaches to multiple attribute (criteria) decision making, this paper makes the following main contributions:

1. We consider a decision problem with the characteristic of multiple attributes and multiple decisions, i.e., there are multiple conditional attributes and multiple decision attributes for the decision problem under consideration, while previous proposals [1,22–26,50,64,66] concentrated on the decision problem with multiple conditional attributes but only one decision attribute.
2. We define two dominance relations over the conditional attributes and decision attributes, respectively. Then we approximate the dominance classes over the decision attributes by using the dominance classes formed by the conditional attributes. Because there is a single decision attribute in previous studies, those proposals approximate equivalence classes [22–26,32] or a fuzzy set (intuitionistic fuzzy set) [27,28,65] over the decision attribute by using the dominance classes formed by the conditional attributes.
3. The decision problem considered in this paper actually is a multiple attribute (criteria) group decision making problem. In this paper, we change the multiple attribute group decision-making into multiple attribute and multiple decision with preference (MA&MD) decision-making. Then we present an approach to multiple decision preference dominance-based rough set decision making. The previous studies for this type of decision problem mainly use the methodology of multiple attribute group decision-making [3,50,54]. As is well known, the weight and the preference of the experts must be aggregated in order to achieve optimal decision making when using the method of group decision making. So, different optimal decision making could be obtained when different weights were given by the
In this paper, we consider a multiple agent conflict decision making problem and then present a MA&MD decision making method to solve it. In order to overcome the limitations of the classical conflict analysis methods, we transform the multiple agent conflict decision making problem into a kind of multiple attribute and multiple decision making problem. Then we establish a multiple decision preference dominance-based rough set model by introducing the multiple decision preference dominance relation. Also, we present a detailed discussion of the properties for the proposed model and develop a decision method for the multiple attribute and multiple decision analysis problem. In order to overcome the limitation that there is no alternative satisfying all decision attributes, we propose a multiple decision $k$-grade preference dominance-based rough set model by defining a $k$-grade dominance relation over decision attributes. Furthermore, we give a variable precision extension of the multiple decision preference dominance-based rough set model and the multiple decision $k$-grade preference dominance-based rough set model by introducing the inclusion degree and precision parameter to a MA&MD decision information system. So, our work extends the ability of the proposed model to deal with practical management decision problems. Additionally, the robustness of the decision results and approach are improved. Finally, we present a new approach to solve the multiple agent conflict decision problem by using the proposed models.

In general, the MA&MD decision making problem considered in this paper can also be discussed using the existing approaches to multiple criteria group decision making [19,33,34,59,62]. Multiple criteria decision analysis, as an important branch of uncertainty decision science, has been studied by many scholars with different theories and approaches. Actually, all the existing studies for the multiple criteria decision problem involve evaluation of the preference for all alternatives. Some problems are essentially humanistic and thus subjective in nature (e.g., human understanding and vision systems). So, a unique or uniform criterion for the evaluation of decision alternatives does not exist. Therefore, every existing decision approach will inevitably have some limitations and advantages. In fact, all the existing theories and approaches to the multiple criteria decision problem have solved different kinds of decision problems effectively. This paper provides a generalized rough set approach to multiple attribute and multiple decision analysis problems. It can be viewed as a new attempt to solve this type of decision problem by using the multiple decision preference dominance-based rough set approach.

Multiple attribute and multiple decision analysis methodology is one of the most important decision making tools in management science, and it has attracted a great deal of attention from both scholars and practitioners in the past few years. This paper studies a kind of multiple agent conflict decision making problem by using the extended dominance-based rough set approach. In this paper, we focus on the basic concepts and models with the decision method for multiple attribute and multiple decision analysis. Further research will develop and revise this approach for more complex multiple decision problems.

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